MA3002 Generell Topologi — Report on Spring 2014 Exam

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June 13, 2014

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Grades

Data

Twenty four students submitted an exam script (ignoring one that was submitted as a formality). The grades are given in Table 1. The average mark was 69.46, a solid C.

Degree Programme	A	B	C	D	E	F
BMAT or MMA (11)	2	2	6	1	0	0
BFY or MTFYMA (5)	1	3	0	1	0	0
MLREAL (5)	1	0	1	1	2	0
Other (3)	0	0	1	0	1	1
All (24)	4	5	8	3	3	1

Table 1: Grades

Verdict

Congratulations to all the students! I have interacted with almost all of you during the term, and feel that every single one of you has a very good talent in, and understanding of, topology!

Grades do not necessarily reflect this, and in an ideal world I would actually prefer that they were not used. Perhaps those students who did not do as well as they had hoped on the day can take some comfort from the knowledge that my opinion of your abilities is very high, and is unaffected by your grade!

I feel that the exam results are excellent. Almost everybody passed, which is a great achievement. It is a credit to the hard work that the students have put in over the term.

To achieve an A on this exam was truly outstanding! The exam scripts of the four of you who did so were superb.

In marking the exams, I have followed the university's grade descriptions, and the university's table for conversion of a mark out of 100 to a grade. A C is a good grade! In addition to the mark, I have in all cases, but with particular deliberation if a mark was close to a grade boundary, thought carefully about the best grade to award from a qualitative point of view.

Though I did not set out purposely to write a harder exam, the students found this year's exam to be harder than last year's. Some students, especially those who have obtained a C, will be disappointed. Nevertheless, I feel that the exam was fair, and that the difficulty level about right. The average mark and grade distribution seem to support this.

Most students felt that the exam was too long. I sympathise with this, and would change the format of the exam in future if I were to teach the course again, to shorten the length a little. My goal is that a student who achieves a good grade should demonstrate competence across the syllabus, at the same time as I like to allow some choice. The exam was written with this in mind. I don't feel that the format used this year and last offers much possibility for cutting the length of the exam whilst meeting these criteria, and it is for this reason that I would use a different format in the future.

Nevertheless, I don't feel that the length of the exam affected most students' grades, and feel that this should not be blown out of proportion. As those students who achieved an A demonstrated, it was possible to write four outstanding answers in the time available. Judging the appropriate level of detail for an answer is a skill which is part of the assessment of the course.

Rather than answering insufficiently many questions, it was errors on questions which should mostly have been mastered for a B that led to a C being awarded. Questions 1 (a), 1 (e), 2 (c), 3 (b), and 3 (d), should not have been too difficult, and those who received a C typically struggled on several of these.

The knot theory question was one which many found to be difficult to complete in the time available. In hindsight, it would have been better to have given the Jones polynomial of the Hopf link in 5 (d), which would have cut down the time slightly. The error in 5 (c) did not help, although, as mentioned in the discussion of Problem 5 below, it does not seem to have had a significant effect on most students' performance.

I do feel that it is fair to assess colouring using links with many crossings, as in 5 (c): it is possible to do this very quickly given sufficient practise. Moreover, it is worth keeping in mind that the aim with asking for four of the five questions to be answered is not that a student can ignore part of the syllabus: the idea is to allow each student to make a judgement as to which questions they feel they will perform best on.

If one knows colouring and calculations with the skein relations well, it is clear that parts 5 (c) and 5 (d) will take some time. One then has to judge whether one feels confident that one can make these calculations quickly enough to justify choosing to answer this problem instead of one of the other four.

I would prefer to emphasise how well I feel that you, as a group, have all done! Congratulations once more!

Individual problems

Problem 1

About half the candidates had difficulties with a). Very few understood what was meant by 'a counterexample': half a mark was deducted if one was not explicitly given.

The part that caused most difficulty, which was expected, was d). Some did not recognise at all the need to demonstrate that the inverse is continuous, in which case very few marks were awarded. Some tried to prove that the inverse is continuous by observing that it is polynomial map, which is not valid, since we do not have the subspace topology from $(\mathbb{R}^2, \mathcal{O}_{\mathbb{R}^2})$. This was one of the questions that I looked at if a candidate's mark was on the A/B boundary: if answered poorly, my inclination was towards a B.

Quite a few also had difficulties with e). This was one of the key questions that I looked at if a candidate's mark was on the B/C boundary: if it was answered poorly, my inclination was to award a C.

For most, b) and c) were fine.

All but one candidate chose this problem as one of their answers. The average mark was 18.35, which corresponds to 73%, so a slightly above average C.

Problem 2

Considering that it was a proof question, a) was answered well. Unfortunately I did not exclude the possibility of quoting the 'continuous surjections preserve connectedness' result from the syllabus, so had to award full marks for a correct answer along these lines, even though it was not what I was looking for. If an actual proof was given as I intended, this was viewed as a positive when making the qualitative decision on the grade.

Question b) was for the most part answered well, but quite a few candidates dropped marks here by not explicitly expressing X as a disjoint union of two sets, or by not justifying why these sets belonged to $\mathcal{O}_X$. Some tried to demonstrate that $(X, \mathcal{O}_X)$ is not path connected which, since connected does not imply path connected, could be awarded almost no marks.

Queston c) was also generally answered quite well, but quite a few did not see how to proceed. Many lost half a mark for not justifying, or not justifying carefully enough, why $\pi^{-1}(\pi(U))$ does not belong to $\mathcal{O}_X$.

Question d) too was typically answered quite well, but most people dropped some marks. The path connectedness approach was typically presented more poorly: the fact that paths can be concatenated and reversed was rarely mentioned, and many did not display any understanding of the formal definition of a path (which could have been conveyed by remarking that a straight line defines a continuous map with I as its source, or something along these lines). In the other approach, many did not mention that one has to apply the 'connected subsets with non-empty intersection' result inductively, or else use a strengthening of this result.

Question e) was generally answered very well, in both the easier version of the question that was actually on the exam, and the harder version that I had in mind (quite a few candidates answered this version).

This problem was overall the best answered. All but one candidate chose to answer it. The overall mark was 20.1, so 80.4%, a lowish B.

Problem 3

This problem was answered poorly, generally. A good performance on it was viewed as a strong positive when making the qualitative decision on the grade.

Performance on a) was about what I expected for a proof question that was not from the syllabus (albeit quite an easy one): several students did give a proof, but quite a lot did not.

Question b) was answered disappointingly poorly. Some did calculate the correct closure, but very many did not.

Question c) was answered very poorly. Some did correctly use the Heine-Borel theorem to see compactness, and then use the fact that compact and Hausdorff implies locally compact. Nobody gave a correct direct proof.

Question d) was answered abysmally, which was very disapppointing: very few calculated it correctly, with many missing one or both of the lines, or the endpoints of the interval segments, and others adding in other points.

I expected e) to be challenging, but even so, very few had any idea how to proceed. There were a handful of correct answers.

I placed particular emphasis on b) and d) especially when making a qualitative decision on the grade for those on the B/C or A/B boundary. Given a weak performance, I inclined to the lower grade. In order to obtain a B, it was important that a very good competence on some questions involving compactness was demonstrated.

This problem was the worst answered of any on the exam. All but two candidates chose this question as one of their answers. The average mark was 13.93, so 55.7%, a low D.

Problem 4

Question a) was generally answered very well. Almost everybody missed, or treated incorrectly, the neighbourhood of the point obtained by gluing the ends of all the edges together. Most also missed some other cases of Hausdorffness. Nevertheless, most achieved a mark of 10 or greater.

Question b) was also generally answered very well.

Question c) was answered excellently.

Although the question was harder than intended, d) was answered well, with many picking up $3\frac{1}{2}$ or 4 marks. Two candidates founds a curve with the correct properties.

This problem was answered by 15 candidates. The average mark was 18.83, so 75.3%, on the B/C boundary.

Problem 5

Almost everybody answered a) correctly.

Question b) was mostly answered very well. Quite a few lost half a mark for failing to clearly distinguish between the sums of the signs in the crossings and the linking number.

Question c) was not typically answered well. For a very few candidates, the error in the question may have caused marks to be lost, but this has been taken into account to the best extent I can when making the qualitative decision on the grade. Overall, the error does not seem to have affected most students, the majority of whom simply did not get far enough with the question, usually due to a lack of time, it seems. Quite a few students lost half a mark or more for failing to justify why $2x \equiv 0 \pmod{m}$ implies that $2 \mid m$, or similar: more marks were lost if an error of this kind was more serious.

Question d) was answered generally quite well. Nobody obtained the correct Jones polynomial, but quite a few just had a small algebraic mistake somewhere, and received $8\frac{1}{2}$. Mistakes with the Reidemeister moves (some thought the trefoil to be the unknot), or incorrect 'breaking up' of a crossing, led to dropped marks for some.

This problem was answered by 15 candidates. The average mark was 17.1, so 68.4%, a low C.