Revision Questions

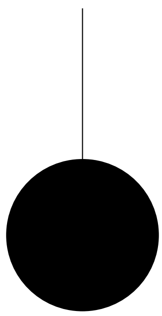
Revision Question 1 — 30/04/14

Let X be the subset of $\mathbb{R}^2$ given by the union of

$$\{(x, y) \mid \|(x-2, y)\| \leq 1\}$$

and

$$\{(2, y) \mid 1 \leq y \leq 3\}.$$



Let $\mathcal{O}_X$ be the subspace topology on X with respect to $(\mathbb{R}^2, \mathcal{O}_{\mathbb{R}^2})$.

a) Is $(X, \mathcal{O}_X)$ compact? Justify your answer. You may quote without proof any results from the syllabus. [5 marks]

b) Is $(X, \mathcal{O}_X)$ Hausdorff? Justify your answer. You may quote without proof any results from the syllabus. [5 marks]

c) Is $(X, \mathcal{O}_X)$ locally compact? You may quote without proof any results from the syllabus. [3 marks]

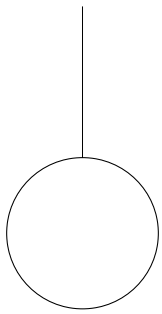
d) Suppose that (x, y) belongs to $\mathbb{R}^2$, and that $\|(x, y)\| \leq 1$. Prove that $D^2 \setminus \{(x, y)\}$, equipped with the subspace topology with respect to $(\mathbb{R}^2, \mathcal{O}_{\mathbb{R}^2})$, is path connected. You may quote without proof any results from the syllabus. [6 marks]

Let Y be the subset of $\mathbb{R}^2$ given by the union of

$$\{(x, y) \mid \|(x-2, y)\| = 1\}$$

and

$$\{(2, y) \mid 1 \le y \le 3\}.$$

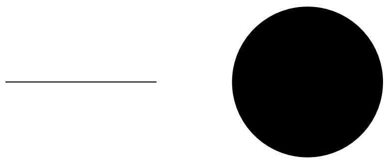


Let $\mathcal{O}_Y$ be the subspace topology on Y with respect to $(\mathbb{R}^2, \mathcal{O}_{\mathbb{R}^2})$.

e) Is $(X, \mathcal{O}_X)$ homeomorphic to $(Y, \mathcal{O}_Y)$? Justify your answer. You may quote without proof any results from the syllabus, except that you may not appeal to the fact that homeomorphisms preserve Euler characteristic. [8 marks]

f) Equip $(X, \mathcal{O}_X)$ with the structure of a Δ-complex, and calculate its Euler characteristic. [6 marks]

Let Z be the union of $\{(x, 0) \mid -2 \le x \le 0\}$ and $\{(x, y) \mid \|(x-2, y)\| \le 1\}$.

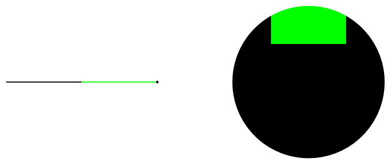


Let $\mathcal{O}_Z$ be the subspace topology on Z with respect to $(\mathbb{R}^2, \mathcal{O}_{\mathbb{R}^2})$. Let $\sim$ be the equivalence relation on Z generated by $(0, 0) \sim (2, 1)$. Let $\mathcal{O}_{Z/\sim}$ be the quotient topology on $Z/\sim$ with respect to $(Z, \mathcal{O}_Z)$. Let A be the union of

$$\{(x, 0) \mid -1 < x < 0\}$$

and

$$\{(x, y) \mid \|(x-2, y)\| \le 1\} \cap \left(\left] \tfrac{3}{2}, \tfrac{5}{2} \right[\times \left] \tfrac{1}{2}, \tfrac{3}{2} \right[\right).$$



Let

$$Z \xrightarrow{\pi} Z/\sim$$

be the quotient map.

g) Does $\pi(A)$ belong to $\mathcal{O}_{Z/\sim}$? Justify your answer. [6 marks]

h) Prove that $(Z/\sim, \mathcal{O}_{Z/\sim})$ is homeomorphic to $(X, \mathcal{O}_X)$. You may quote without proof any results from the course. [8 marks]