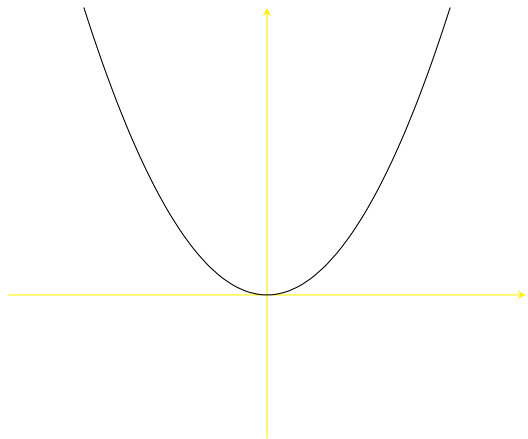
Revision Question 2 — 01/05/14

Let X be the subset of $\mathbb{R}^2$ given by

$$\left\{(x, x^2) \mid x \in \mathbb{R}\right\}.$$



Let $\mathcal{O}_X$ be the subspace topology on X with respect to $(\mathbb{R}^2, \mathcal{O}_{\mathbb{R}^2})$.

a) Prove that $(X, \mathcal{O}_X)$ is homeomorphic to $(\mathbb{R}, \mathcal{O}_{\mathbb{R}})$. You may quote without proof any facts from the course. [5 marks]

b) Give an example of a subset Y of $\mathbb{R}^2$ such that the following hold.

   i) Y is not closed in $\mathbb{R}^2$ with respect to $\mathcal{O}_{\mathbb{R}^2}$.

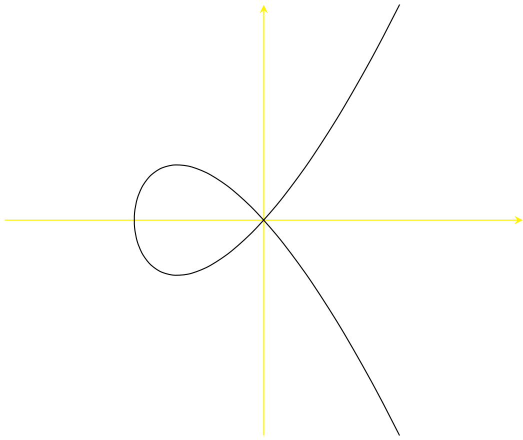
   ii) $(Y, \mathcal{O}_Y)$ is homeomorphic to $(\mathbb{R}, \mathcal{O}_{\mathbb{R}})$, where $\mathcal{O}_Y$ is the subspace topology on Y with respect to $(\mathbb{R}^2, \mathcal{O}_{\mathbb{R}^2})$.

   You do not need to prove that your set Y satisfies i) and ii). [5 marks]

c) What is the closure in $\mathbb{R}^2$ with respect to $\mathcal{O}_{\mathbb{R}^2}$ of your set Y of part b)? You do not need to give a proof. [3 marks]

Let Z be the subset of $\mathbb{R}^2$ given by

$$\left\{(x^2 - 1, x^3 - x) \mid x \in \mathbb{R}\right\}.$$



Let $\mathcal{O}_Z$ be the subspace topology on Z with respect to $(\mathbb{R}^2, \mathcal{O}_{\mathbb{R}^2})$.

d) Is $(Z, \mathcal{O}_Z)$ homeomorphic to $(\mathbb{R}, \mathcal{O}_{\mathbb{R}})$? Justify your answer. You may quote without proof any results from the syllabus. [6 marks]

e) Prove that $(Z, \mathcal{O}_Z)$ is connected. You may quote without proof any results from the syllabus. [5 marks]

f) Prove that $(-3, 0)$ is not a limit point of Z in $\mathbb{R}^2$ with respect to $\mathcal{O}_{\mathbb{R}^2}$. [4 marks]