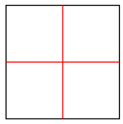
Revision Question 9 — 08/05/14

Let A be the subset of I^2 given by $\left(\frac{1}{2} \times I\right) \cup \left(I \times \frac{1}{2}\right)$.



a) Prove that A is closed in I^2 with respect to $\mathcal{O}_{I^2}$. [4 marks]

b) Give two different proofs that A is a connected subset of I^2 with respect to $\mathcal{O}_{I^2}$. You may quote without proof any results from the course. [6 marks]

c) Let $\mathcal{O}_{I^2 \setminus A}$ be the subspace topology on $I^2 \setminus A$ with respect to $(I^2, \mathcal{O}_{I^2})$. What is the connected component of $(\frac{1}{4}, \frac{3}{4})$ in $I^2 \setminus A$ with respect to $\mathcal{O}_{I^2 \setminus A}$? Justify your answer. [6 marks]

Let $\sim$ be the equivalence relation on I^2 given by $x \sim y$ if both x and y belong to $I^2 \setminus A$.

d) Prove that $(I^2/\sim, \mathcal{O}_{I^2/\sim})$ is not Hausdorff. [5 marks]

e) Find a subset B of $I^2/\sim$ which is compact with respect to $\mathcal{O}_{I^2/\sim}$, but which is not closed in $I^2/\sim$ with respect to $\mathcal{O}_{I^2/\sim}$. Justify why your set B has these properties. You may quote without proof any results from the course. [6 marks]

f) Is there a compact subset of I^2 which is not closed? Justify your answer. You may quote without proof any results from the course. [4 marks]

Let $\approx$ be the equivalence relation on I^2 given by $x \approx y$ if both x and y belong to $\partial_{(\mathbb{R}^2, \mathcal{O}_{\mathbb{R}^2})} I^2$. Let

$$I^2 \xrightarrow{\pi} I^2/\approx$$

be the quotient map.

g) Let $\mathcal{O}_{\pi(A)}$ be the subspace topology on $\pi(A)$ with respect to $(I^2/\approx, \mathcal{O}_{I^2/\approx})$. Let $\mathcal{O}_A$ be the subspace topology on A with respect to $(I^2, \mathcal{O}_{I^2})$. Is $\left(\pi(A), \mathcal{O}_{\pi(A)}\right)$ homeomorphic to $(A, \mathcal{O}_A)$? Justify your answer. [6 marks]