NTNU – Trondheim
Norwegian University of
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Department of Mathematical Sciences

Examination paper for **MA1301 Number theory**

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Examination date: Monday 6th October 2014

Examination time (from–to): 15:15 – 16:45

Permitted examination support material: D: No printed or hand-written support material is allowed. Permitted calculators: Hewlett Packard HP30S, Citizen SR-270X, Citizen SR-270X College, Casio fx-82ES PLUS.

Other information:
Answer all four problems. Each problem is worth 5 marks. The possible marks for each part is given in brackets.

If you cannot solve a part of a problem after having tried a while, move on and come back later to it instead: don't spend too much time on each part.

If you cannot solve a part of a problem, write down in any case as much as you can regarding how you would go about solving it.

You can appeal to an assertion in a part of a problem in the rest of the problem, even if you have not shown that the assertion is true.

Good luck!

Language: English

Number of pages: 2

Number pages enclosed: 0

Checked by:

Date Signature

Problem 1 The sequence of Fibonacci numbers is defined recursively as follows.

(1) The first Fibonacci number is 1.

(2) The second Fibonacci number is 1.

(3) Assume that the first m Fibonacci numbers have been defined, where m is a given natural number such that $m \geq 2$. We then define the $(m+1)$-st Fibonacci number to be the sum of the m-th Fibonacci number and the $(m-1)$-st Fibonacci number.

For any natural number r, let us denote the r-th Fibonacci number by u_r. Then (3) says that

$$u_{m+1} = u_m + u_{m-1}.$$

a) Write out the first five Fibonacci numbers. [1 poeng]

b) Let n be a natural number. Prove that

$$u_2 + u_4 + u_6 + \cdots + u_{2n} = u_{2n+1} - 1.$$

[4 poeng]

Problem 2

a) Let n be an integer. Assume that there is an integer k such that $n = 5k + 3$. Show that there then is an integer m such that

$$n^2 = 5m + 4.$$

HInt: Use that $9 = 5 + 4$ in the course of your answer. [1 poeng]

b) Let n be an integer. Show that there is an integer m such that one of the following assertions is true:

(1) $n^2 = 5m$;
(2) $n^2 = 5m + 1$;
(3) $n^2 = 5m + 4$;

[4 poeng]

Problem 3

a) Use Euclid's algorithm to find an integer solution to the equation

$$295x - 126y = 27.$$

[4 poeng]

b) Is the following assertion true or false: the equation

$$295x - 126y = c$$

has an integer solution for every integer c? Justify your answer. [1 poeng]

Problem 4

a) Explain why $7 \equiv -1 \pmod 8$. [1 poeng]

b) Show without calculating that $7^{33} \equiv -1 \pmod 8$. [1 poeng]

c) Show that

$$3^{77} + 3 \cdot 7^{33}$$

is divisible by 8, without calculating the sum. [3 poeng]